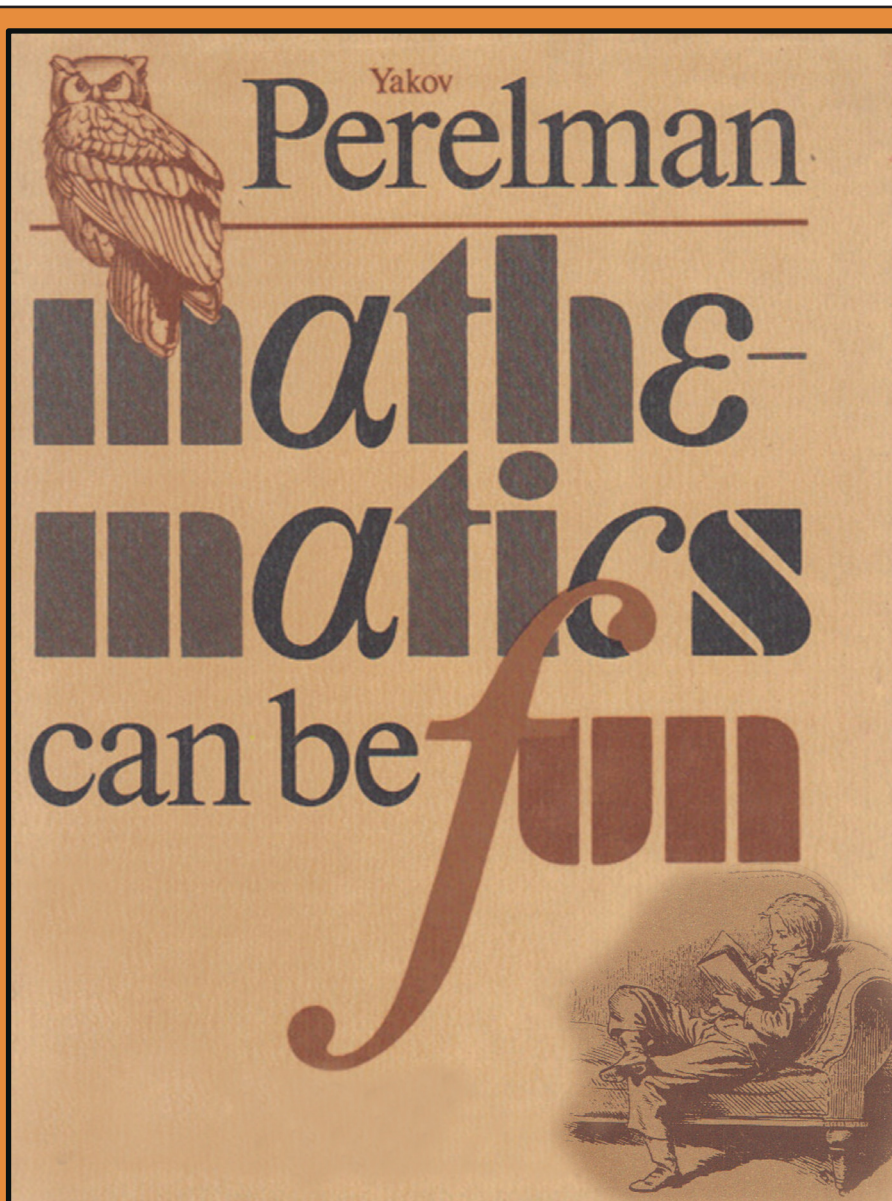




About the Author

In 1913 in Russian bookshops appeared a book by the outstanding educationalist Yakou Isidorovich Perelman entitled *Physics for Entertainment*. It struck the fancy of the young who found in it the answers to many of the questions that interested them.

Physics for Entertainment not only had an interesting layout, it was also immensely instructive.

In the preface to the 11th

edition Perelman wrote: "The main objective of *Physics for Entertainment* is to arouse the activity of scientific imagination, to teach the reader to think in the spirit of the science of physics and to create in his mind a wide variety of associations of physical knowledge with the widely differing facts of life, with all that he normally comes in contact with."

Physics for Entertainment was a best seller.

Y. I. Perelman was born in 1882

in the town of Byelostok (now in Poland). In 1909 he obtained a diploma of forester from the St. Petersburg Forestry Institute. After the success of *Physics for Entertainment* Perelman set out to produce other books, in which he showed himself to be an imaginative popularizer of science. Especially popular were *Arithmetics for Entertainment*, *Mechanics for Entertainment*, *Geometry for Entertainment*, *Astronomy for Entertainment*, *Figures for Fun*, *Physics Everywhere*, and *Tricks and Amusements*. Today these books are known to every educated person in the Soviet Union.

He has also written several books on interplanetary travel (*Interplanetary Journeys*, *On a Rocket to Stars*, *World Expanses*, etc.).

The great scientist K. E. Tsiolkovsky thought highly of the talent and creative genius of Perelman. He wrote of him in the preface to *Interplanetary Journeys*: "The author has long been known by his popular, witty and quite scientific works on physics, astronomy and mathematics, which are moreover written in a marvelous language and are very readable."

Perelman has also authored a number of textbooks and articles in Soviet popular science magazines.

In addition to his educational, scientific and literary activities, he has also devoted much time to editing. So he was the editor of the magazines *Nature and People* and *In the Workshop of Nature*.

Perelman died on March 16, 1942, in Leningrad.

Many generations of readers have enjoyed Perelman's fascinating books, and they will undoubtedly be of interest for generations to come.

Preface

To read and enjoy this book written by Ya. Perelman will suffice to possess a modest knowledge of mathematics, i.e. A knowledge of the rules of arithmetic and elementary geometry. Very few problems require the ability of forming and solving equations, and the simplest at that.

The coverage may be, is quite diverse: the subject ranges from a motley collection of conundrums and mathematical stunts to useful practical problems on counting and measuring. The author has done everything to make his book as fresh as possible, avoiding repetition of all that has already appeared in his other works (Tricks and Amusements, Interesting Problems, etc.). The reader will find a hundred or so brain-teasers that have not been included in earlier books. Chapter 6 - "Number Giants" - is adapted from one of the author's earlier pamphlets, with four new stories added.

01. Brain - Teasers for Lunch

It was raining..... We had just sat down for lunch at our holiday home when one of the guests asked us whether we would like to hear what had happened to him in the morning.

Everyone assented, and he began.

1. A Squirrel in the Glade

"I had quite a bit of fun playing hide-and-seek with a squirrel," he said. "You know that little round glade with a lone birch in the center? It was on this tree that a squirrel was hiding from me. As I emerged from a thicket, I saw its snout and two bright little eyes peeping from behind the trunk. I wanted to see the little animal, so I started circling round along the edge of the glade, mindful of keeping the distance in order not to scare it. I did four rounds, but the little cheat kept backing away from me, eyeing me suspiciously from behind the tree. Try as I did, I just could not see its back."

"But you have just said yourself that you circled round the tree four times" one of the listeners interjected.

"Round the tree, yes, but not round the squirrel."

"But the squirrel was on the tree, wasn't it?"

"So it was."

"Well, that means you circled round the squirrel too."

"Call that circling round the squirrel when I didn't see its back?"

"What has its back to do with the

whole thing? The squirrel was on the tree in the centre of the glade and you circled round the tree. In other words, you circled round the squirrel."

"Oh no, I didn't. Let us assume that I'm circling round you and you keep turning, showing me just your face. Call that circling round you?"

"Of course, what else can you call it?"



"You mean I'm circling round you though I'm never behind you and never see your back?"

"Forget the back! You're circling round me and that's what counts. What has the back to do with it?"

"Wait. Tell me, what's circling round anything? The way I understand it, it's moving in such a manner so as to see the object I'm moving around from all sides. Am I right, professor?" He turned to an old man at our table.

"Your whole argument is essentially on about a word," the professor replied. "What you should do first is agree on the definition of 'circling'. How do you understand the words 'circle round an object'? There are two ways of understanding that. First, it's moving round an object that is in the centre of a circle. Secondly, it's moving round an object in such a way as to see all its sides. If you insist on the first meaning, then you walked round the squirrel four times. If it's the second that you hold to, then you did not walk round it at all. There's really no ground for an argument here, that is, if you two speak the same language and understand words in the same way."

"All right, I grant there are two meanings. But which is the correct one?"

"That's not the way to put the question. You can agree about anything. The question is, which of the two meanings is the more generally accepted? In my opinion, it's the first, and here's why. The sun, as you know, does a complete revolution in a little more than 25 days....."

"Dose the sun revolve?"

Of course, it does, like the earth

around its axis. Just imagine, for instance, that it would take not 25 days, but 365 1/4 days, i. e. a whole year, to do so. If this were the case, the earth would see only one side of the sun, that is, only its 'face'. And yet, can anyone claim that the earth does not revolve round the sun?"

"Yes, now it's clear that I circled round the squirrel after all."

"I've a suggestion, comrades!" one of the company shouted. "It's raining now, no one is going out, so let's play riddles. The squirrel riddle was a good beginning. Let each think of some brain-teaser."

"I give up if they have anything to do with algebra or geometry." a young woman said.

"Me too," another joined in.

"No, we must all play, but we'll promise to refrain from any algebraical or geometrical formulas, except, perhaps, the most elementary ones. Any objections?"

"None!" the others chorussed.

"Let's go."

"One more thing. Let the professor be our judge."

2. School - Groups

"We have five extra-curricular groups at school," a schoolboy began. "They're fitters', joiners', photographic, chess, and choral groups. The fitters group meets every other day, the joiners' every third day, the photographic every fourth day, the chess every fifth day and the choral every sixth day. These five groups first met on January 1 and thenceforth meetings were held according to schedule. The question is, how

many times did all five groups meet on the same day in the first quarter (January 1 excluded)?"

"Was it a Leap Year?"

"No."

"In other words, there were 90 days in the first quarter."

"Right."

"Let me add another question," the professor broke in. "How many days were there when none of the groups met in the first quarter?"

"So there's a catch to it! There won't be another day when all the five groups meet, and there won't be one day when none meet. That's clear!"

"Why?"

"Don't know. But I've a feeling there's a catch."

"Comrades!" said the man who had suggested the game. "We won't reveal the results now. Let's have more time to think them. The professor will announce the answers at supper."

3. A Log Problem

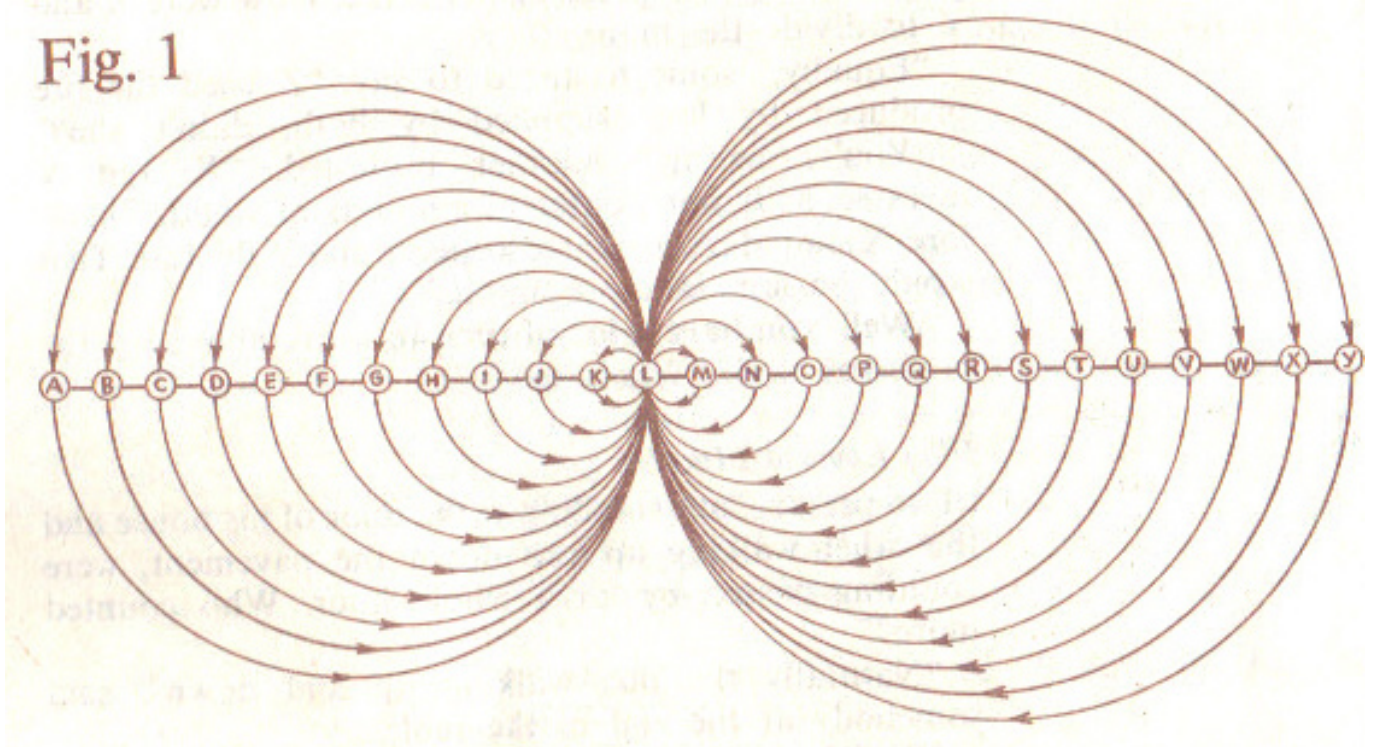
"It happened at a summer cottage. A household problem, you might call it. The cottage is shared by three persons - let's call them X, Y, and Z. It's an old house with an old-fashioned cooking stove. X put three logs into the stove, Y added five and Z, who had no firewood, paid them eight kopecks as her share. How did X and Y divide the money?"

"Equally," some hastened to say.

"Z used the fire produced by logs supplied by both, didn't she?"

"You're wrong," another protested. "X and Y invested, as it were, different amounts of wood. Therefore, X should receive

Fig. 1



three kopecks and Y the rest. That would be fair, in my opinion.”
 “Well, you have a lot of time to think about it,” the professor said.
 “Who’s next?”

4. Who Counted More?

“Two persons, one standing at the door of his house and the other walking up and down the pavement, were counting passers-by for a whole hour. Who counted more?”
 “Naturally the one walking up and down,” said somebody at the end of the table.
 “We’ll know the answer at supper,” the professor said.
 “Next!”

5. Grandfather and Grandson

“In 1932 I was as old as the last two digits of my birth year.

When I mentioned this interesting coincidence to my grandfather, he surprised me by saying that the same applied to him too. I thought that impossible”
 “Of course that’s impossible,” a young woman said.
 “Believe me, it’s quite possible and grandfather proved it too. How old was each of us in 1932?”

6. Railway Tickets

“I’m a railway ticket seller,” said the next person, a young lady.
 “people think this job is easy. They probably have no idea how many tickets one has to sell, even at a small station. There are 25 stations on my line and I have to sell different tickets for each section up and down the line. How many different kinds of tickets do you think I have at my station?”
 “Your turn next,” the professor said to a pilot.

7. A Helicopter’s Flight

“A helicopter took off from Leningrad and flew north. After five hundred kilometres it turned and flew 500 kilometres east. After that it turned south and covered another 500 kilometres. Then it flew 500 kilometres west, and landed. The question is, where did it land: west, east, north, or south of Leningrad?”
 “That’s an easy one,” someone said. “Five hundred steps forward, 500 to the right, 500 back, and 500 to the left, and you’re naturally back where you’d started from!”
 “Easy? Well than, where did the helicopter land?”
 “In Leningrad, of course. Where else?”
 “Wrong!”
 “Then I don’t understand.”
 “Yes, There’s some catch to this puzzle,” another joined in. “Didn’t

the helicopter land in Leningrad?”
“Won’t you repeat your problem?”
The pilot did. The listeners looked at each other. “All right,” the professor said. “We have enough time to think about the answer. Let’s have the next one now.”

8. Shadow

“My puzzle,” said the next man, “is also about a helicopter. What’s broader, the helicopter or its perfect shadow?”

“Is that all?”

“It is.”

“Well, then. The shadow is naturally broader than the helicopter: sun-rays spread fanlike, don’t they?”

“I wouldn’t say so,” another interjected. “Sun-rays are parallel to each other and that being so, the helicopter and its shadow are of the same size.”

“No, they aren’t. Have you ever seen rays spreading from behind a cloud? If you have, you’ve probably noticed how much they spread. The shadow of the helicopter must be considerably bigger than the helicopter itself, just as the shadow of a cloud is bigger than the cloud itself.”

Then why is it that people say that sun-rays are parallel to each other? Seamen, astronomers, for instance.”

The Professor put a stop to the argument by asking the next person to go ahead with his conundrum.

Answers 1 to 8

1. The squirrel puzzle was explained earlier, so we’ll pass on to the next.

2. We can easily answer the first question: how many times did all the five group meet on the same day in the first quarter (January 1 excluded) by finding the least common multiple of 2,3, 4, 5 and 6. That isn’t difficult. It’s 60. Therefore, the five will all meet again on the 61st day - the fitters’ group after 30 two-day intervals, the joiners’ after 20 three-day intervals, the photographic after 15 four-day intervals, the chess after 12 five-day intervals and the choral after 10 six-day intervals. In other words, they can meet on the same day only once in 60 days. And since there are 90 days in the first quarter, it means there can be only every other day on which they will meet.

It is much more difficult to find the answer to the second question: how many days are there when none of the groups meet in the first quarter? To find that, it is necessary to write down all the numbers from 1 to 90 and then cross out all the days when the fitters’ group meets: e.g. 1, 3, 5, 7, 9, etc. After that one must cross out the joiners’ group’s days: e.g. 4, 7, 10, etc. When all the photographic, chess and choral groups’ days have also been crossed out the numbers that remain are the days when there is no group meeting.

Do that and you’ll see that there are 24 such days-eight in January, i.e. 2, 8, 12, 14, 18, 20, 24 and 30, seven in February and nine in March

3. It is not right to think, as many do, that eight kopecks were paid for eight logs, a kopeck for a log. The money was paid for one-third

of eight logs because the fire they produced was used equally by all three. Consequently, the eight logs were estimated to be worth 8×3 , i.e. 24 kopecks, and the price of a log was therefore three kopecks. It is now easy to see how much was due to each. Y’s five logs were worth 15 kopecks and since she had used 8 kopecks worth of fire, she would have to receive $15 - 8$, i. e. 7 kopecks. X would have to receive 9 kopecks, but if you subtracted the 8 kopecks due from her for using the stove, you would see that she had to receive $9-8$, i. e. 1 kopeck.

4. Both of them counted the same number of passers-by. While the one who stood at the door counted all those who passed both ways, the one who was walking counted all the people he met going up and down the pavement.

There is another way of putting it. When the man who was walking and counting the passers-by returned for the first time to the man who was standing at the door, they had counted the same number of passers-by - all those passing by the standing man encountered the walking man either on the way there or back. And each time the one who was walking was returning to the one who was standing he counted the same number of passers-by. It was the same at the end of the hour, when they met for the last time and told each other the final number.

5. At first it may seem that the problem is incorrectly worded, that both grandfather and grandson are of the same age. We shall soon see that there is nothing wrong with the problem.

It is obvious that the grandson was born in the 20th century. Therefore, the first two digits of his birth year are 19 (the number of hundreds). The other two digits added to themselves equal 32. The number therefore is 16: the grandson was born in 1916 and in 1932 he was 16.

The grandfather, naturally, was born in the 19th century. Therefore, the first two digits of his birth year are 18. The remaining digits multiplied by 2 must equal 132. The number sought is half of 132, i. e. 66. The grandfather was born in 1866 and in 1932 he was 66.

Thus, in 1932 the grandson and the grandfather were each as old as the last two digits of their birth years.

6. At each of the 25 stations passengers can get tickets for any of the other 24. Therefore, the number of different tickets required is: $25 \times 24 = 600$. There may also be round-trip tickets. If so, the number doubles and there are then 1 200 different tickets.

7. There is nothing contradictory in this problem. The helicopter did not fly along the perimeter of a square. It should be borne in mind that the earth is round and that the meridians converge at the poles (Fig.2.). Therefore, when the helicopter flew the 500 kilometres along the parallel 500 kilometres north of Leningrad, it covered more degrees eastward than it did when it was returning along the Leningrad latitude. As a result, the helicopter completed its flight east of Leningrad.

How many kilometres away? That can be calculated. Figure 3 shows the route taken by the helicopter: ABCDE. N is the North Pole where meridians AB and DC meet. The helicopter first flew 500 kilometres northward, i. e. along meridian AN. Since the degree of a meridian is 111 kilometres long, the 500-kilometre-long arc of the meridian is equal to $500 \div 111 \approx 405'$. Leningrad lies on the 60th parallel. B, therefore, is on $600 + 640.5$. The helicopter then flew eastward, i. e. along the BC parallel, covering 500 kilometres. The length of one degree of this

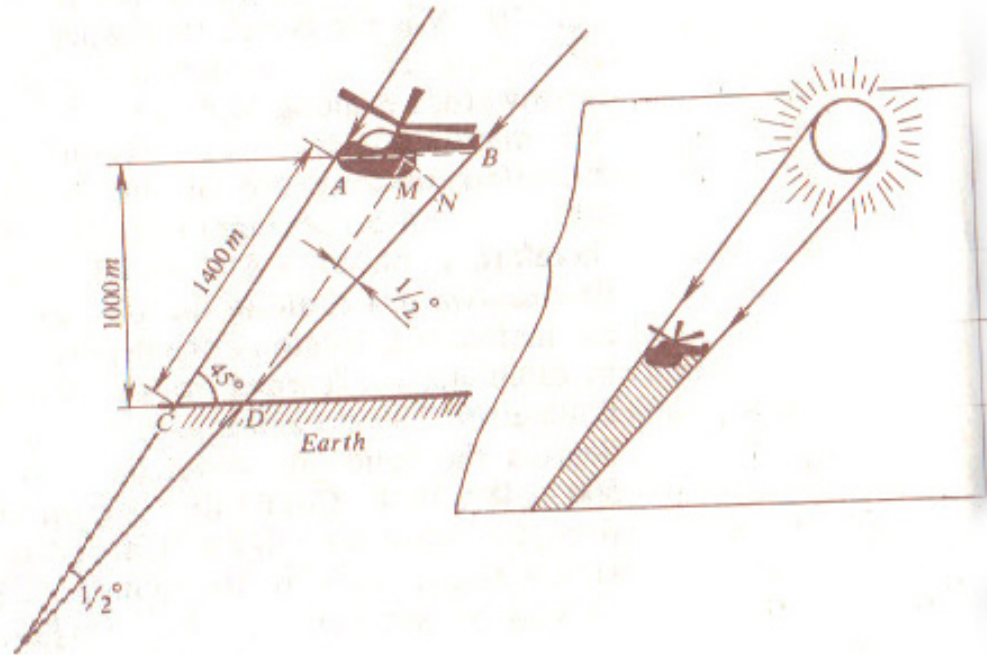
parallel may be calculated (or learned from tables); it is equal to 48 kilometres. Therefore, it is easy to determine how many degrees the helicopter covered in its eastward flight: $500 \div 48 \approx 1004'$. Continuing, the aircraft flew southward, i. e. along meridian CD, and, having covered 500 kilometres, returned to the Leningrad parallel. Thence the way lay westward, i.e. along DA; the 500 kilometres of this way are obviously less than the distance between A and D. there are as many degrees in AD as in BC, i.e. $1004'$. But the length of 1° at the 60th parallel equals 55.5 kilometres. Therefore, the distance between A and D is equal to $55.5 \times 10.4 = 577$ kilometres. We see thus that the helicopter could not have very well landed in Leningrad: it landed 77 kilometres away, on Lake Ladoga.

8. In discussing this problem, the people in our story made several mistakes. It is wrong to say that the rays of the sun spread fanlike noticeably. The Earth is so small in comparison to its distance from

Fig. 2



Fig. 3



the Sun that sun-rays falling on any part of its surface radiate at an almost absolutely imperceptible angle; in fact, rays may be said to be parallel to each other. We occasionally see them spreading fanlike (for instance, when the sun is behind a cloud). This, however, is nothing but a case of perspective.

Parallel lines, as they recede from the station point, always appear to the eye to meet far away at a point, e.g. railway tracks or a long avenue.

But the fact that sun-rays fall to the ground in parallel beams does not mean that the perfect shadow of a helicopter is as broad as the helicopter itself. Figure 3 shows that the perfect shadow of the helicopter narrows down in space on the way to the surface of the earth and that, consequently, the shadow the helicopter casts should be narrower than the helicopter,

i.e. CD is shorter than AB.

It is quite possible to compute the difference, provided, of course, we know at what altitude the helicopter is flying. Let us assume that the altitude is 100 metres. The angle formed by lines AC and BC is equal to the angle from which the sun is seen from the earth.

We know that this angle is equal to $1/2^\circ$. On the other hand, we know that the distance between the eye and any object seen from an angle of $1/2^\circ$ is equal to the length of 115 diameters of this object. Hence, section MN (the section seen from the surface of the earth at an angle of $1/2^\circ$) should be $1/115$ of AC. Line AC is longer than the perpendicular distance between point A and the surface of the earth. If the angle formed by the sun's rays and the surface of the earth is equal to 45° , then AC (given the altitude of the helicopter is 100 metres) is

approximately 140 metres long and section MN is consequently equal to $140 \div 115 = 1.2$ metres. But then the difference between the helicopter and its shadow, i.e. section MB, is bigger than MN (1.4 times, to be exact), because angle MBD is almost equal to 45° . Therefore, MB is equal to 1.2×1.4 , and that gives us almost 1.7 metres.

All this applies to the perfect – black and sharp – shadow of the helicopter, and not to the penumbra, which is weak and hazy.

Incidentally, our computation shows that if instead of a helicopter we had a small balloon about 1.7 metres in diameter, there would be no perfect shadow. All we would see would be a hazy penumbra.